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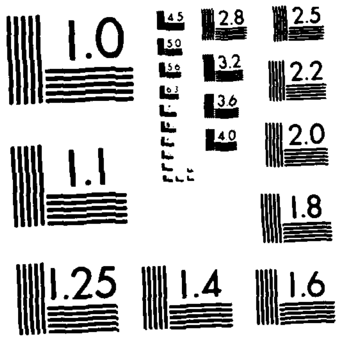

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STRESS RATES IN CONTINUUM MECHANICS AND COMPUTER CODES

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10 April 1986

Technical Report

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The concept of an objective (frame-indifferent) stress rate is introduced. The behavior of several objective stress rates in a material undergoing simple extension is discussed. The response of a hypoelastic material in simple shear, in both the elastic and elastic/plastic regimes, is evaluated, and it is concluded that for materials with low yield strength, such as metals and geologic materials, use of the Jaumann stress rate is justified.

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SECTION 1
STRESS RATES IN CONTINUUM MECHANICS AND COMPUTER CODES

The necessity of paying careful attention to stress rates in the theory of mechanics of a continuous medium is illustrated by a simple example: Suppose a body undergoes rigid body motion (rotation and translation) without stretching, i.e.,

$$D_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) = 0 \tag{1}$$

where $\underline{x}$ = position, $\underline{v}$ = velocity and $\underline{\underline{D}}$ is the rate-of-deformation tensor. Then, at every point in the body, in general, the time derivative of the stress tensor is constant only in a frame of reference attached to the body, i.e., a rotating frame. In other words,

$$\dot{\underline{\underline{\sigma}}} \equiv \frac{D\underline{\underline{\sigma}}}{Dt} \neq 0 \tag{2}$$

In this case of rigid body motion, the quantity

$$\underline{\underline{\sigma}}_J \equiv \dot{\underline{\underline{\sigma}}} - \underline{\underline{W}}\,\underline{\underline{\sigma}} + \underline{\underline{\sigma}}\,\underline{\underline{W}} \tag{3}$$

actually is = 0; here $\underline{\underline{W}}$ is the spin tensor:

$$W_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right) \tag{4}$$

The quantity $\dot{\underline{\underline{\sigma}}}_J$ is called the Jaumann stress rate, and is one of an infinite family of stress rates which satisfy the principle of objectivity, or frame indifference. In addition to these objective stress rates, other objective tensor quantities are $\underline{\underline{D}}$, scalars, and the so-called Rivlin-Ericksen tensors; examples of non-frame-indifferent tensor quantities are $\underline{\underline{W}}$, $\underline{v}$ and $\dot{\underline{\underline{\sigma}}}$. For a discussion of frame-indifference, see [1]. The objective stress rate typically appears in a constitutive relation of the form

$$\overset{\circ}{\sigma}_{ij} = C_{ijk\ell}\left(D_{k\ell} - D^{p}_{k\ell}\right) \tag{5}$$

where $\underline{\underline{\underline{\underline{C}}}}$ is the elastic constitutive tensor and $\underline{\underline{D}}^{p}$ the plastic strain rate.

Stress rates which are closely related to the Jaumann rate and which have appeared in the literature are the Cotter-Rivlin rate:

$$\overset{\circ}{\underline{\underline{\sigma}}}_{CR} \equiv \overset{\circ}{\underline{\underline{\sigma}}}_{J} + \underline{\underline{D}}\,\underline{\underline{\sigma}} + \underline{\underline{\sigma}}\,\underline{\underline{D}} \quad , \tag{6}$$

the Oldroyd rate:

$$\overset{\circ}{\underline{\underline{\sigma}}}_{O} \equiv \overset{\circ}{\underline{\underline{\sigma}}}_{J} - (\underline{\underline{D}}\,\underline{\underline{\sigma}} + \underline{\underline{\sigma}}\,\underline{\underline{D}}) \tag{7}$$

and the Truesdell rate:

$$\overset{\circ}{\underline{\underline{\sigma}}}_{T} \equiv \overset{\circ}{\underline{\underline{\sigma}}}_{O} + \underline{\underline{\sigma}}\ \mathrm{tr}\ \underline{\underline{D}} \tag{8}$$

These rates are defined in terms of tensor quantities evaluated in the current material configuration (one point tensors), and are easy to incorporate in either an Eulerian or Lagrangian computer code. An objective stress rate which is gaining popularity in Lagrangian codes is the Green-Naghdi rate:

$$\overset{\circ}{\underline{\underline{\sigma}}}_{GN} = \dot{\underline{\underline{\sigma}}} - \underline{\underline{\Omega}}\,\underline{\underline{\sigma}} + \underline{\underline{\sigma}}\,\underline{\underline{\Omega}} \tag{9}$$

where

$$\underline{\underline{\Omega}} \equiv \dot{\underline{\underline{R}}}\,\underline{\underline{R}}^{T} \tag{10}$$

and $\underline{\underline{R}}$ is the proper-orthogonal rotation tensor in the polar decomposition of the deformation gradient:

$$\underline{\underline{F}} = \underline{\underline{R}}\,\underline{\underline{U}} \tag{11}$$

where

$$F_{ij} = \partial x_i / \partial X_j \tag{12}$$

Here $\underline{X}$ is the position of the particle, currently at $\underline{x}$, in the undeformed body; $\underline{\underline{F}}$ is a so-called two point tensor. It is easily seen that the evaluation of $\overset{\circ}{\underline{\underline{\sigma}}}_{GN}$ requires "keeping books" on the original position of each material point in the continuum, and while feasible in Lagrangian codes would be considerably more complicated in an Eulerian context.

In [2] it is shown that of the Jaumann, Oldroyd, Cotter-Rivlin and Truesdell rates, only the Jaumann rate gives a linear stress-strain response for a hypoelastic body of grade zero undergoing simple extension, i.e., for

$$\underline{\underline{D}} = \begin{bmatrix} \dot{\eta} & 0 & 0 \\ 0 & \dot{\eta} & 0 \\ 0 & 0 & \dot{\epsilon} \end{bmatrix} \tag{13}$$

where η and ϵ are functions of time (see Figure 1). In addition, it is shown in [3] that of the Jaumann, Oldroyd, Cotter-Rivlin and Truesdell stress rates, only the Jaumann rate has the property that when it vanishes the stress invariants are stationary.

However, it is shown in [4] that the Jaumann rate suffers a serious deficiency: consider a linear elastic medium undergoing pure rectilinear shear (plane strain), with

$$\underline{\underline{D}} = \begin{bmatrix} 0 & a/2 & 0 \\ a/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{14}$$

$$\underline{\underline{W}} = \begin{bmatrix} 0 & a/2 & 0 \\ -a/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{15}$$

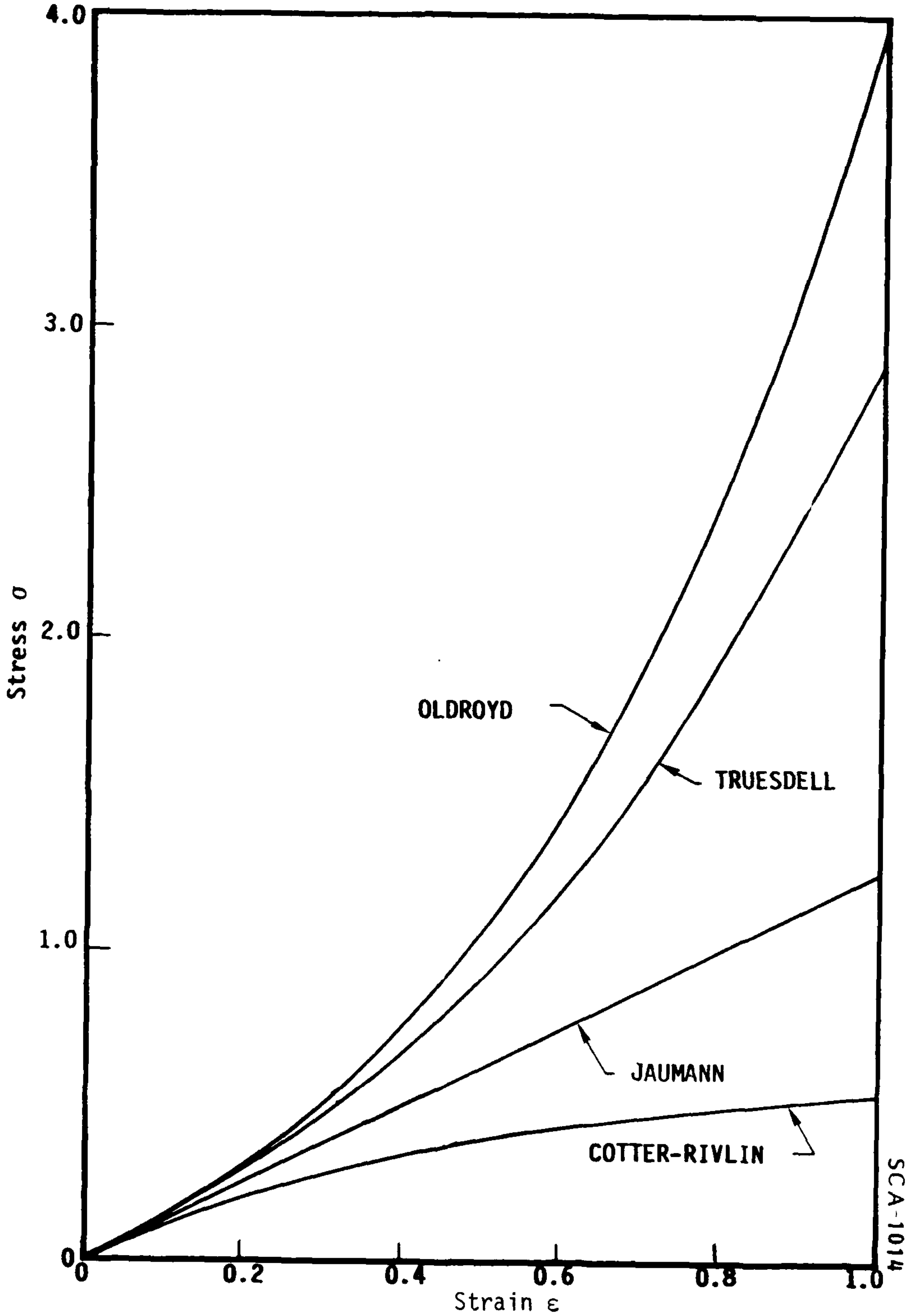

Figure 1. The constitutive relations for a hypoelastic body of grade zero undergoing simple extension for Poisson ratio = 1.4).

and obeying the constitutive relation

$$\overset{\circ}{\underline{\underline{\sigma}}}_J = \lambda \, \underline{\underline{I}} \; \mathrm{tr} \, \underline{\underline{D}} + 2\mu \, \underline{\underline{D}} \tag{16}$$

where λ and μ are the Lame´ constants and $\underline{\underline{I}}$ is the identity tensor, with stress and strain equal zero initially, and a = constant. The governing equations are then

$$\dot{\sigma}_{11} - a \, \sigma_{12} = 0 \tag{17a}$$

$$\dot{\sigma}_{12} - \frac{1}{2} a \left(\sigma_{22} - \sigma_{11}\right) = \mu a \tag{17b}$$

$$\dot{\sigma}_{22} + a \, \sigma_{12} = 0 \tag{17c}$$

$$\dot{\sigma}_{33} = 0 \tag{17d}$$

which have the solution (for the above initial conditions)

$$\sigma_{12} = \mu \sin at \tag{18a}$$

$$\sigma_{11} = \mu(1 - \cos at) \tag{18b}$$

$$\sigma_{22} = - \sigma_{11} \tag{18c}$$

$$\sigma_{33} = 0 \tag{18d}$$

The sinusoidal variation of stress is clearly physically unreasonable. On the other hand, it is shown in [4] that the use of $\overset{\circ}{\underline{\underline{\sigma}}}_{GN}$ instead of $\overset{\circ}{\underline{\underline{\sigma}}}_J$ in (16) gives a monotonic behavior of stress for this problem (Figure 2). This fact has led to the implementation of the Green-Naghdi rate in Lagrangian codes such as NIKE2D, DYNA2D and HONDO.

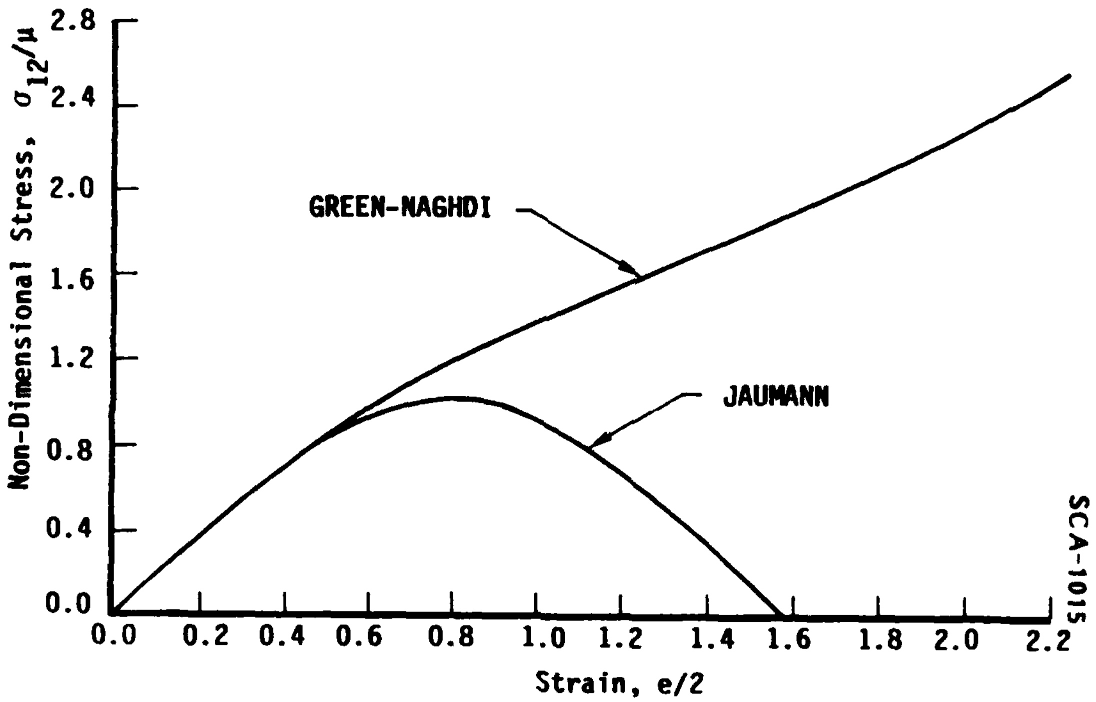

Figure 2. A comparison between the shear stress on a hypo-elastic material in a simple, rectilinear shear using the Green-Naghdi and the Jaumann stress rates.

Consider the above model problem of rectilinear shear with a stress rate of the form

$$\overset{\circ}{\underline{\underline{\sigma}}} = \dot{\underline{\underline{\sigma}}} - \underline{\underline{W}}\,\underline{\underline{\sigma}} + \underline{\underline{\sigma}}\,\underline{\underline{W}} + \alpha(\underline{\underline{\sigma}}\,\underline{\underline{D}} + \underline{\underline{D}}\,\underline{\underline{\sigma}}) + \beta\,\underline{\underline{\sigma}}\,\mathrm{tr}\,\underline{\underline{D}} \tag{19}$$

where α and β are constants. Note that the Cotter-Rivlin, Oldroyd and Truesdell rates are included in this form. For this particular problem the governing equations are

$$\dot{\sigma}_{11} + (\alpha - 1)\, a\, \sigma_{12} = 0 \tag{20a}$$

$$\dot{\sigma}_{12} + \frac{1}{2}\left[(\alpha + 1) a\, \sigma_{11} + (\alpha - 1) a\, \sigma_{22}\right] = \mu a \tag{20b}$$

$$\dot{\sigma}_{22} + (\alpha + 1) a\, \sigma_{12} = 0 \tag{20c}$$

$$\dot{\sigma}_{33} = 0 \tag{20d}$$

Thus

$$\ddot{\sigma}_{12} = (\alpha - 1)(\alpha + 1) a^2\, \sigma_{12} \tag{21}$$

For the case $(\alpha - 1)(\alpha + 1) = 0$ the solution is

$$\sigma_{12} = \mu\, a\, t \tag{22a}$$

$$\sigma_{11} = 0 \qquad (\alpha = 1) \tag{22b}$$

$$\phantom{\sigma_{11}} = a^2 \mu t^2 \qquad (\alpha = -1)$$

$$\sigma_{22} = -a^2 \mu t^2 \qquad (\alpha = 1) \tag{22c}$$

$$\phantom{\sigma_{22}} = 0 \qquad (\alpha = -1)$$

For the case $(\alpha - 1)(\alpha + 1) > 0$ the solution is

$$\sigma_{12} = \left\{\mu/\left[(\alpha - 1)(\alpha + 1)\right]^{1/2}\right\}\sinh\left\{\left[(\alpha - 1)(\alpha + 1)\right]^{1/2} at\right\} \tag{23a}$$

$$\sigma_{11} = \left[\mu/(\alpha + 1)\right]\left[1 - \cosh\left\{\left[(\alpha - 1)(\alpha + 1)\right]^{1/2} at\right\}\right] \tag{23b}$$

$$\sigma_{22} = \left[\mu/(\alpha - 1)\right]\left[1 - \cosh\left\{\left[(\alpha - 1)(\alpha + 1)\right]^{1/2} at\right\}\right] \tag{23c}$$

and for the case $(\alpha - 1)(\alpha + 1) < 0$ the solution is

$$\sigma_{12} = \left\{\mu/\left[-(\alpha - 1)(\alpha + 1)\right]^{1/2}\right\} \sin\left\{\left[-(\alpha - 1)(\alpha + 1)\right]^{1/2} at\right\} \tag{24a}$$

$$\sigma_{11} = \left[\mu/(\alpha + 1)\right]\left[1 - \cos\left\{\left[-(\alpha - 1)(\alpha + 1)\right]^{1/2} at\right\}\right] \tag{24b}$$

$$\sigma_{22} = \left[\mu/(\alpha - 1)\right]\left[1 - \cos\left\{\left[-(\alpha - 1)(\alpha + 1)\right]^{1/2} at\right\}\right] \tag{24c}$$

In all cases, $\sigma_{33} = 0$. All of the above solutions (22-24) have terms which are exponentially growing, sinusoidal (non-monotonic) or of order t^2, all of which seem physically unreasonable.

However, the departure from "reasonable" behavior in those solutions occurs only after the deformation is very large, i.e., after the shear strain becomes of order unity. For metals and geologic materials, plastic failure occurs long before this point, so it is useful to investigate this problem in an elastic-plastic context. Suppose the governing equations are the above in the elastic regime, with the Jaumann stress rate, and, in the plastic regime, the associated flow rule

$$\underline{\underline{D}}^P = \Lambda\, \underline{\underline{S}} \tag{25}$$

where $\underline{\underline{D}}^p$ is the plastic strain rate:

$$\underline{\underline{D}} = \underline{\underline{D}}^e + \underline{\underline{D}}^p \tag{26}$$

where $\underline{\underline{D}}^e$ is the elastic strain rate, and

$$\underline{\underline{\sigma}}_J = \lambda\ \underline{\underline{I}}\ \mathrm{tr}\ \underline{\underline{D}}^e + 2\mu\ \underline{\underline{D}}^e \tag{27}$$

with yield function

$$f = \frac{1}{2} S_{ij}\ S_{ij} - \frac{1}{3} Y^2 \tag{28}$$

where Y is the (constant) stress difference in compression for a von Mises material and S is the stress deviator:

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\ \sigma_{kk} \tag{29}$$

This is the von Mises failure surface and the Prandtl-Reuss flow rule. In the (initial) elastic solution, $J_2' = 1/2\ S_{ij}\ S_{ij}$ reaches a maximum at dimensionless time at = π, and we suppose plastic failure occurs before this point. Subsequently, as long as the material is on the yield surface, the governing equations are

$$\dot{\sigma}_{11} - a\ \sigma_{12} = -2\mu\ \Lambda\ S_{11} \tag{30a}$$

$$\dot{\sigma}_{12} + \frac{1}{2}\ a\left(\sigma_{11} - \sigma_{22}\right) = \mu a - 2\mu\ \Lambda\ S_{12} \tag{30b}$$

$$\dot{\sigma}_{22} + a\ \sigma_{12} = -\ 2\mu\ \Lambda\ S_{22} \tag{30c}$$

$$\dot{\sigma}_{33} = 2\mu\ \Lambda\ (S_{11} + S_{22}) \tag{30d}$$

Adding (30a), (30c) and (30d), we obtain

$$\begin{aligned}\dot{\sigma}_{11} + \dot{\sigma}_{22} + \dot{\sigma}_{33} &= \mathrm{tr}\ \dot{\sigma} \\ &= 0\end{aligned} \tag{31}$$

and since at the end of the elastic solution, tr $\underline{\underline{\sigma}} = 0$, we have also for the plastic solution tr $\underline{\underline{\sigma}} = 0$, so that $\underline{\underline{\sigma}} = \underline{\underline{S}}$.

Now

$$J_2' = S_{11}^{\ 2} + S_{22}^{\ 2} + S_{12}^{\ 2} + S_{11}\, S_{22} \tag{32}$$

$$= \frac{1}{3}\, Y^2$$

so that

$$2S_{11}\mathring{S}_{11} + 2S_{22}\mathring{S}_{22} + 2S_{12}\mathring{S}_{12} + S_{11}\mathring{S}_{22} + S_{22}\mathring{S}_{11} = 0 \tag{33}$$

which gives

$$-\,4\mu\,\Lambda\left(S_{11}^{\ 2} + S_{22}^{\ 2} + S_{12}^{\ 2} + S_{11}\,S_{22}\right) + 2\mu\,a\,S_{12} = 0 \tag{34}$$

so that

$$\Lambda = 3a\,S_{12}/2Y^2 \tag{35}$$

Also,

$$\mathring{S}_{11} + \mathring{S}_{22} = -\,2\mu\,\Lambda\,(S_{11} + S_{22}) \tag{36}$$

$$= \frac{3a\,\mu}{Y^2}\,(S_{11} + S_{22})\,S_{12}$$

Since $S_{11} = -\,S_{22}$ at the end of the elastic solution, (36) is satisfied thereafter if $S_{11} = -\,S_{22}$, so that the governing equations in the plastic flow regime become

$$\mathring{S}_{11} - a\,S_{12} = -\,\frac{3a\,\mu}{Y^2}\,S_{11}\,S_{12} \tag{37a}$$

$$\mathring{S}_{12} + a\,S_{11} = \mu a - \frac{3a\,\mu}{Y^2}\,S_{12}^{\ 2} \tag{37b}$$

$$S_{22} = -\,S_{11} \tag{37c}$$

$$S_{33} = 0 \tag{37d}$$

If a steady state exists for the equations (37), it is

$$S_{11} = \frac{Y^2}{3\mu} \tag{38a}$$

$$S_{12} = \left[\frac{Y^2}{3}\left(1 - \frac{Y^2}{3\mu^2}\right)\right]^{1/2} \tag{38b}$$

so that for a steady state to exist, we must have

$$Y \leq 3^{1/2}\ \mu \tag{39}$$

The character of the solution is as follows: if: $Y \geq 12^{1/2}\ \mu$ the elastic solution (18) persists forever. If $Y < 12^{1/2}\ \mu$, the elastic solution holds until dimensionless time

$$at_o = \cos^{-1}\left(1 - \frac{Y^2}{6\mu^2}\right) \tag{40}$$

at which time

$$S_{11}(t_o) = \frac{Y^2}{6\mu} \tag{41a}$$

$$S_{12}(t_o) = Y\left[\left(1 - Y^2/12\mu^2\right)\big/3\right]^{1/2} \tag{41b}$$

If $Y \leq 3^{1/2}\ \mu$, the solution asymptotes to (38); otherwise the solution oscillates between the elastic and plastic flow regimes. This has been verified by numerical solution of (37). Figure 3 shows the solution for the case $Y = \mu = 100$ kb; here $S_{12}(\infty)/S_{12}(t_o) = 0.85$. In the case $Y = \mu/10$ (see Figure 4), $S_{12}(\infty)/S_{12}(t_o) = 0.9987$. In all cases where a steady state exists, $S_{11}(\infty) = 2S_{11}(t_o)$.

These results indicate that for real materials with $Y << \mu$, the Jaumann stress rate should be adequate.

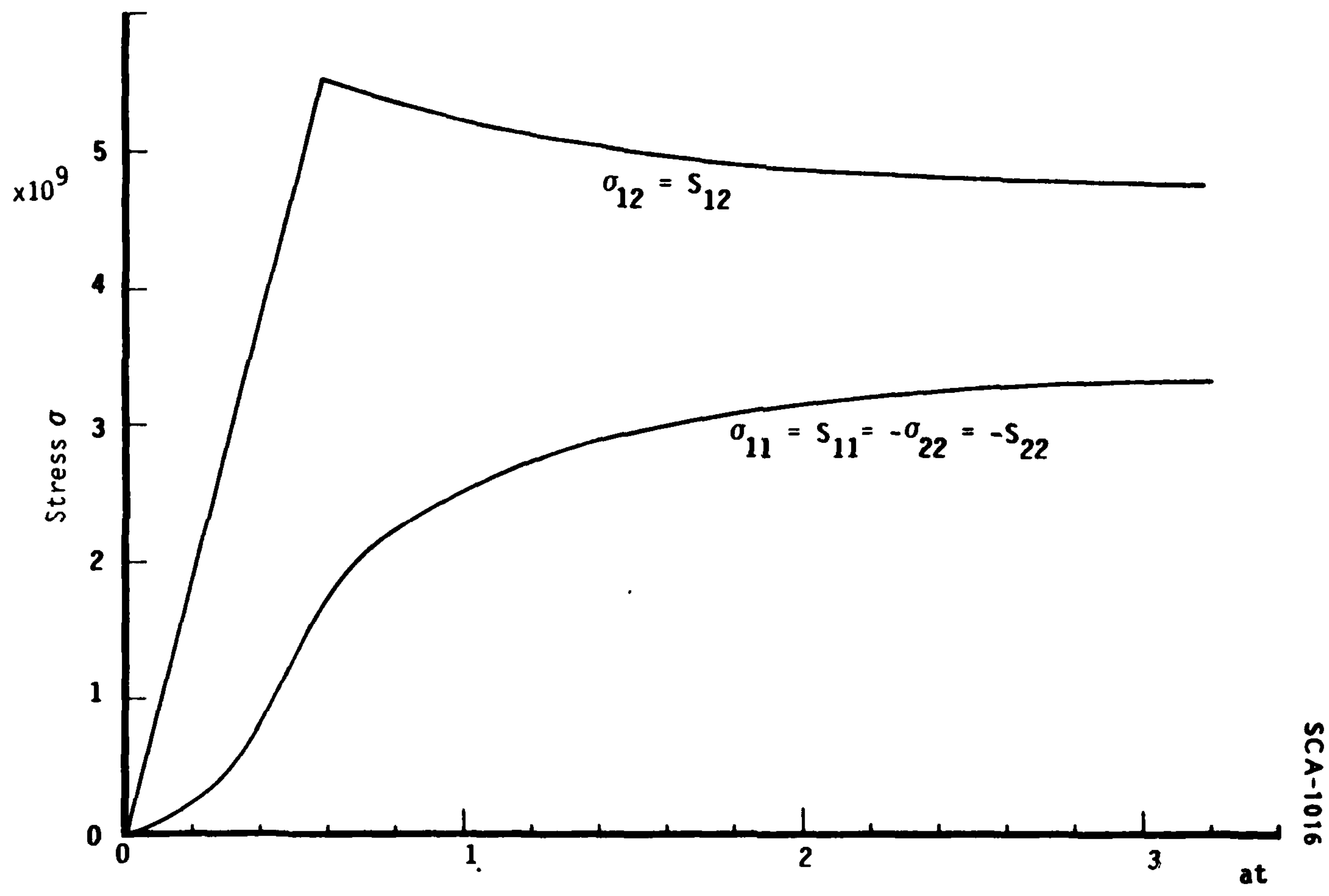

Figure 3. Elastic/plastic solution for $Y = \mu$: Stress vs. dimensionless time.

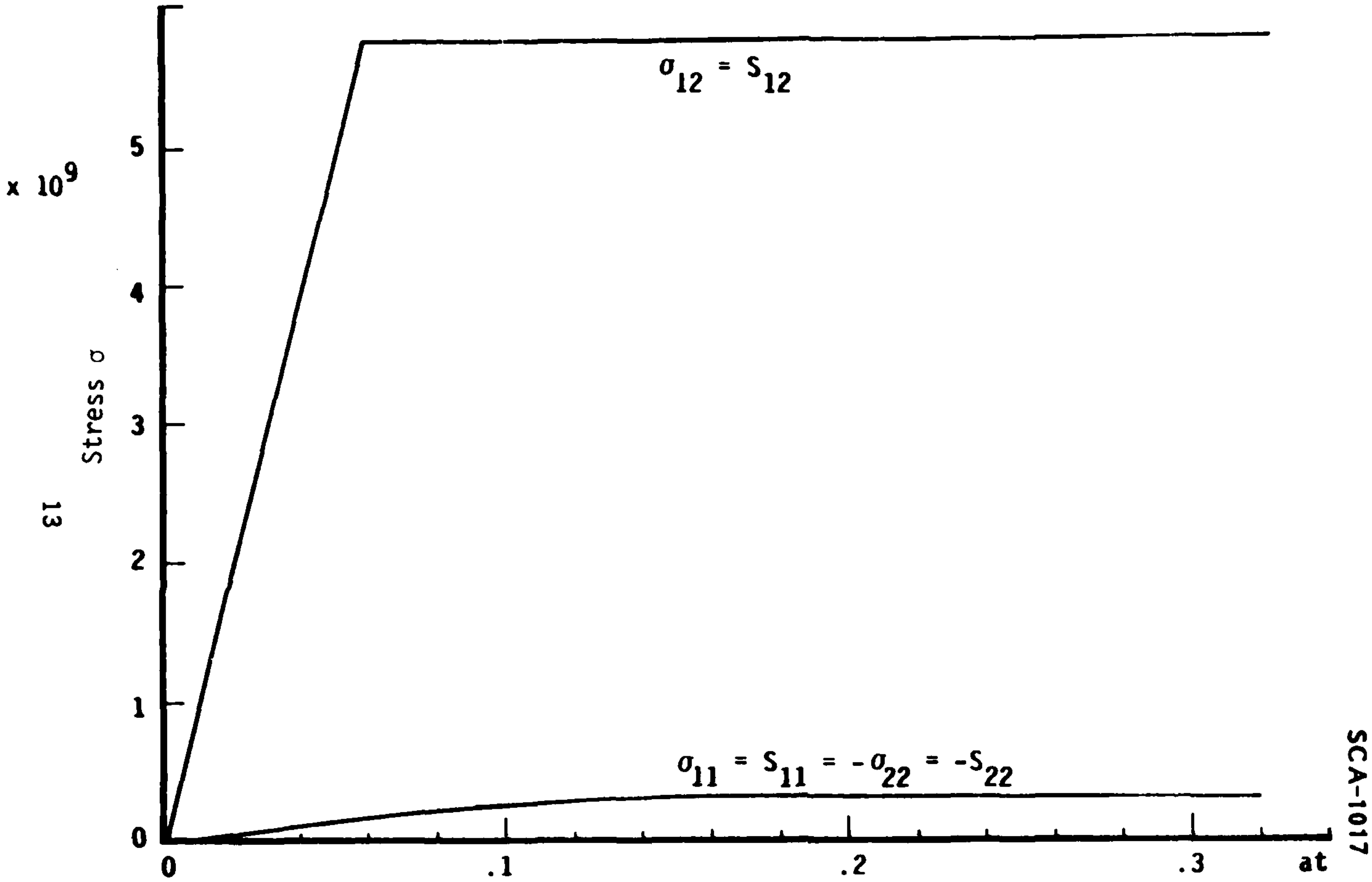

Figure 4. Elastic/plastic solution for $Y = \mu/10$: Stress vs. dimensionless time.

SECTION 2
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